ANALYSIS " COMPETITION "—SECOND " PROBLEM "

A REPORT on the result of the " problem " set for solution in ANALYSIS for January 1952, together with the four best entries, appears in this issue. With fifteen entries, and much interest shown by readers who did not compete, the new venture has won sufficient support to justify a further attempt.

The second " problem " is, therefore, set by Professor Austin Duncan-Jones of Birmingham, Chairman of the Analysis Committee, and is as follows :—

' What is the difference between saying how you feel and showing by your words how you feel ?'

Entries (still not more than 600 words long), accompanied by a stamped, addressed envelope, should be sent to **The Editor of Analysis by Tuesday September 30th, 1952.** *No entries should be sent to Professor Duncan-Jones.* Contributors may submit entries either under their own names or a pseudonym. A report, with names of successful entrants and, if possible, their entries, will be published in ANALYSIS for December 1952.

THE EDITOR.

REPORT ON ANALYSIS " PROBLEM " No. I

" WHAT SORT OF ' IF ' IS THE ' IF ' IN ' I CAN IF I CHOOSE ' ? "

By J. L. AUSTIN

FIFTEEN entries were received, seven from Great Britain, three from the United States, three from Australia, one from Belgium and one from Palestine. Nearly all contained at least one idea of interest, but none, to my mind, carried both conviction and clarity to the end. The limit of 600 words was not observed very strictly : perhaps it was too low, yet in most entries words were wasted.

Only one entrant maintained outright what has, I think, been pretty regularly taken for granted, that this is an " ordinary conditional " or " causal " *if*, and that " I can " is somehow the same as " I shall ", which is interpreted as a forecast. Several others, however, produced variations on this theme. While I do not know the answer to my own problem, I feel certain that this is wrong.

Several entrants suggested, as at least one possible interpretation, that "I can if I choose" is analytic—somewhat similar to "I can if I ought" taken in the sense of the dogma "Ought implies can". But has anyone ever used the expression in this way?

It was noticeable how few were the "sorts of *if*" which entrants considered to be already sufficiently known. Apart from the two (vague enough) referred to above, the only ones mentioned were the concessive *if* = *even if* and the *if* of conditional promises.

Interest was shown in several promising lines of attack on the problem. What is the "truth table" (if any!) for this *if*? What are the linguistic occasions for the use of this expression? What is the effect of putting, *e.g.*, 'shall' for 'can' or 'try' for 'choose'? But none of these lines was pursued very thoroughly.

The four best entries were those of "Cuckoo", Brian Ellis, Douglas Gasking, and G. M. Matthews (alphabetical order). I eliminate Douglas Gasking because he discusses "I *could* if I *chose*", which biases his answer: nevertheless his concept of the "reasonably possible" is a useful one, and I hope the Editor may find room to print him. The other three all get off to good starts, though their conclusions, especially the most definite (G. M. Matthews), will perhaps not be found so powerful.

Good wishes and witticisms, notably from Messrs. Henle, Killalea and Wessel, were gratefully received.

Magdalen College, Oxford.

"I CAN IF I CHOOSE"

By Cuckoo

IF I say "I can if I choose" (C) I do not claim to be able to on condition that I choose, as in "I can if I hurry" (H) I claim to be able to on condition that I hurry. The 'if' in *C* is not, as we may call the 'if' in *H*, an 'if' of condition.

The belief that it is comes—blind faith apart—from confusing the conditions for the proper use of C with what C is used to say. I am entitled to use C when I am in a position to do what I claim to be able to do; e.g., if I properly use "I can if I choose" to assert my right to squander my fortune on orchids and horse-races, no legal or other difficulties must bar the way.

I must not be bound, for instance, by the conditions of my father's will to use the money for something else. I must be free to do what I say I can do—a different sense of 'free', incidentally, from that opposed to 'determined'. Now, there is a temptation to express this *contextual* requirement, viz., that there should be nothing to impede me should I choose to do what I say I can do, by saying that my doing it depends only on my choosing. There is then the further temptation to construe this, in its turn, as expressing for C something similar to what "my doing it depends on my hurrying" expresses for *H*, viz., to think that "if I choose" says that my choosing is the only condition necessary for my doing, and thus that the 'if' in C is one of condition. This, however is a mistake.

A clue can be got to the function of "if I choose" in C by comparing, first, two other sentences: "I can if I hurry" (H) and "I can if I choose to hurry" (J). J does not differ from H in that it has a new or an extra condition; in both H and J I claim to be able to on the same condition, viz., that I hurry. J and H differ in what they imply about whether I shall hurry, though neither H nor J implies that I shall or that I shall not hurry. The difference between them is this: I should use J if there is the spoken or unspoken presumption that I shall hurry, and I wished to rebut this presumption. J indicates that it must not be taken for granted that I shall hurry, and implies that there is a possibility that I may not. Since J rebuts a presumption that will have been made in a social context it is well-suited to unsociable performances like sulking, showing resentment, standing on one's dignity and so on.

Now consider "I can if I choose" (C), in which "if I choose" has a similar uncooperative "Don't you assume I will —I may not" function. Because it would be used not merely to rebut a presumption, but in an open clash, C has great emotional force. To say "I can if I choose" to the friend who remonstrates against your dissipation of your patrimony—"But you can't just *waste* your father's hard-won fortune!"—is to disregard his entreaty: by saying "I can" you remind him that you are within your rights (legal, etc.) in doing so; by adding "if I choose" you warn him that, what's more, there is the possibility that you may exercise this right. This warning or threatening or cautionary . . . aspect of C (which it is will depend on the particular context in which C is used) is especially prominent because the 'if'-clause comes last. We may note that C does not imply that you will do what you are exhorted not to do (cf. "I can and will") but only that you *may* do it.

We could call the ' if ' in C the ' if ' of possibility-in-opposition, because it opposes :

" Don't! " by

" I will, possibly."

London University.

" I CAN, IF I CHOOSE "

By BRIAN ELLIS

WE might say that two sentences were of the same sort if they exhibited the same grammatical form, or if their use or some feature could be characterized in the same way. The answer that we would give to the question would in this way depend on the context in which it occurred. We might say that two words were of the same sort if they could be interchanged in most of the contexts in which they appeared. Sometimes a word serves different purposes in different contexts. Under these circumstances we can speak of different sorts of ' know ' or ' understand ' or whatever the word may be. Such a difference in function is revealed by effecting small changes of a similar kind in each of the two contexts. If the word be a noun we may simply insert an adjective, or if it be a verb, an adverb. If nonsense is produced in one case, but sense in the other, we say that the word has a different function in one sentence from what it has in the other. Thus we might substitute ' when ' for ' if ' in " I can if I choose ", and in " I'll come if it rains ". The first substitution is seen to produce " I can when I choose " and this is just an idiomatic way of saying the same thing, but the second becomes " I'll come when it rains ", a sentence with quite a different use. Or again if we substitute the word ' although ' for ' if ' we get. " I'll come although it is raining ", a perfectly significant sentence, but making this substitution in " I can if I choose " we get the meaningless sentence " I can although I choose ". A third and most revealing way of showing that there is a difference is to translate " I can if I choose " by " I can, but whether or not I choose is another matter ". The two sentences have the same meaning. However, making a similar construction with " I'll come if it rains " we get " I'll come, but whether or not it will rain is another matter ". The two sentences are seen to have quite different meanings. This shows that the " I choose " does not state the conditions under

which the "I can" might be true. In most 'if' sentences the 'if' clause gives the conditions under which the 'then'-clause might be true. But to show that there is a difference in logic between the 'if' of "I can if I choose" and the 'ifs' of conditional sentences is not to say what sort of 'if' the 'if' is, but rather to say what sort of 'if' it is not. We can answer the question if we can say what the function of the 'if I choose'-clause is in the sentence in question. The "I can" is supposedly used as a claim of ability. The "if I choose" does not state conditions under which the claim might be true; it is added as a protection against the inference that because one can, one should. "I can if I choose" can be translated "I can, but it is still open for me to say whether or not I will' or 'I can, but this does not say that I will." The "if I choose" is used as a protection against faulty inference. This 'if'-clause like others is concerned with inference and therein lies the reason for the word 'if' at all, but whereas the 'conditional-if' clauses give the conditions under which the 'then' clause can be inferred to be true, this "non-committal" if-clause is included as a bar to faulty inference.

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"I COULD IF I CHOSE"

By DOUGLAS GASKING

LET "It is technically possible for me to do x" mean that my doing x is physically possible and that I don't lack any necessary skill, instrument or opportunity. E.g., it is technicaly possible for me to lift a 100 lb. sack of potatoes, to get the exhaust manifold off the cylinder-block (I have the skill and the spanner), and to sell my house for £3,000 (there are buyers waiting).

Let "It is reasonably possible for me to do x" mean that I have no good reason, moral or other, for refusing to do x if requested or for refraining if I want to. E.g., it is reasonably possible for me to accept your invitation for Wednesday (I have no other engagements and am not so frantically busy that I couldn't spare the time). It is not reasonably possible for me to take two months holiday immediately. (It would mean resigning my job or taking French leave and getting the sack. And it

would be idiotic to do either just for the sake of an immediate holiday, even if I wanted one).

If an action is both technically and reasonably possible I say " I can " or " I could " do it. Otherwise I usually just say " I can't " or " I couldn't " whether the impossibility is technical or reasonable (or both). E.g., I may simply reply " Sorry, I couldn't " to your request for me to translate three pages of Russian for you by to-morrow, whether it is because I know no Russian or because I am so busy that I could not do your job without neglecting my other work or sitting up unreasonably late when I'm dead tired. The reply " I couldn't ", which leaves it open which sort of impossibility it is, is often all that is needed. For often all my hearer wants to know is whether there is some chance of my doing it or no chance, and is not interested in whether the impossibility is technical or reasonable.

But if, for some reason, I wish to make it plain that my " couldn't " reports a reasonable impossibility only, and that the action *is* technically possible, I do so by saying " I could, if I chose ". E.g., " Why don't you come with us on our yacht-cruise to the Barrier Reef? " " But I can't. I've got a job to do, and I can't just walk out like that! " " Oh, come on! Why not just drop everything and come? You *could*, you know! " " I suppose I could if I chose. But really, you know, it's quite impossible. For one thing . . . etc. "

To say " I could if I chose " is thus a way of saying that something is technically but not reasonably possible. (In a similar way " I couldn't " leaves it open whether the impossibility is technical or reasonable, whereas " I couldn't, even if I chose " emphasizes that the impossibility is a technical one). The force of adding " if I chose " to " I could " is that of adding " But, mind you, it is only technically possible, and not also reasonably possible, as you might have supposed from my saying " I could".

I have given a real-life context for " I could if I chose ". No conclusions about the use of this phrase should be drawn from the case of the philosopher who mutters " I could lift my hand if I chose ". For he is not considering whether he could or not, but only pretending to.

What sort of " if "? The " if " of entailment and of causal implication records a connection between protasis and apodosis. So also, though a connection of a different sort, does the " if " of conditional promises. Standing apart from this big family of connective " ifs " is one well-known non-connective specimen, namely the concessive " if ", as in " If he's not very original it is at least, a careful piece of work ". Our " if " is of a special

sort, belonging, with the concessive " if " to the non-connective family. For my ability to do x does not in any sense depend on my choosing to do it.

University of Melbourne.

"I CAN IF I CHOOSE"

By G. M. MATTHEWS

UNLIKE the apodosis of the ordinary conditional, the apodosis in this sentence is logically independent. If I can, I can whether I choose or not. My ability is not conditional upon my choice, and this can be ascertained from consideration of the meaning of the words alone. This independence of the apodosis appears to characterize all sentences of this " (ability verb) if (preference verb) " pattern, e.g. " He knows quite well how to behave if he wants to ". " You could have done that if you had wished to ".

Prima facie there would seem to be other apodoses which are logically independent but which do not contain ability verbs, e.g. " There is a train leaving at 7.30 if that is not too early for you ". " There is the Roger Fry exhibition if you would care to go." However, these appear to be elliptical, and may be more fully expressed by sentences which do contain verbs expressing ability. " There is a train leaving at 7.30 which you can catch if it is not too early for you." " There is the Roger Fry exhibition to which you can go if you would care to ". I therefore suggest that this peculiarity of independence of the apodosis is to be explained by some peculiarity of what I have called ability and preference verbs.

These verbs appear to be distinctive in that they make a claim, and of rather a special sort. In saying, " I can do *The Times* Crossword " I make a claim that, provided I have a copy of the *Times*, that the crossword has not been done, etc., etc., and if I want to, I will succeed in doing it. Under these conditions the claim may be established or the right to it refused. If the conditions are not fulfilled, the claim may be taken on trust or disbelieved. " I can do it, there is nothing to prevent me, and I want to do it, but I'm not going to " is, I suggest, a contradiction. " I want to do the crossword, and I can do it, but I haven't a pencil " is not a contradiction in this usage.

" I can do *The Times* Crossword " may, however, be used to

make a stronger claim. It may be practically equivalent to "I can and nothing prevents me . . . " In this usage the absence of a pencil, or any other impediment, serves to rebut the claim. Sometimes these two usages may become muddled. " I cannot do it without a pencil, but I can do it all the same " would be one way of voicing the puzzle produced.

The case of ability verb plus preference verb is an instance of the second usage. It is presupposed that all other conditions are favourable. " No you cannot, because your parents would not allow it " is a sufficient rebuttal. One is claiming that if this one condition is fulfilled, e.g. that I choose, then the crucial experiment will take place.

" I can if I choose " has this further peculiarity. As Aristotle says, we choose to do what we think is in our power, although we may wish for the impossible. There is an element of redundancy, since " I choose to do x " already implies " I am able to do x ", and thus the claim is made in both parts of the sentence. If the claim fails, both clauses must be withdrawn.

Thus I have attempted to argue that the ' if ' in " I can if I choose " serves to introduce the specification of the test conditions under which alone the claim expressed by " I can " may be established, or the right to it refused.

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RECENT CRITICISMS OF RUSSELL'S ANALYSIS OF EXISTENCE

By ALAN DONAGAN

LORD RUSSELL'S definition of existence (as he calls it) has an equivocal reputation : although many accept it as a secure part of philosophical analysis, there are murmurs against it. In repudiating extravagant pretensions of the theory of incomplete symbols and logical constructions excess is tempting. Two criticisms of Lord Russell's definition, one at least of which is well-known, appear to me to suffer from it.

In her paper *The Philosopher's Use of Analogy* first, published fourteen years ago, and recently republished in A. G. N. Flew's *Logic and Language*, Miss Margaret Macdonald attacks Lord Russell's definition at its root. From Kant's discovery that existence is not a predicate, Lord Russell argued that, in such

sentences as 'Lions exist', the *grammatical* subject 'lions' does not stand for any object or objects: *logically* it is not a subject, but a predicate. What the sentence 'Lions exist' means might be more clearly expressed by 'Something is a lion', although such a sentence would not ordinarily be used. To this Miss Macdonald offers the following objection.

> 'According to Russell, "There are men" or "A man exists" means "The property of being human belongs to something" or "x is human" is sometimes true, and "All horses neigh" means "Whatever is equine neighs" or "x is equine and neighs" is always true.[1] According to this they are similar in type though one is more complex than the other since it is a compound of two or more propositional functions. But we might be inclined to ask (assuming that "man" here means "man or woman, i.e. human being") what is it that has the property of being human except a man? What is it that is equine except a horse which also neighs? That this view did lead to a more sophisticated attempt to find ultimate subjects for predicates seems to be shown by Russell's doctrine that the ultimate elements of facts are logical atoms named by logically proper names unadulterated by any description' (*Logic and Language*, pp. 89–90).

Lord Russell's analysis is to be rejected because it 'leads to' the absurd doctrine that existential propositions have more 'ultimate' subjects than the horses, men, and gods of plain men. Miss Macdonald, we may observe, does not claim that Lord Russell's analysis necessarily committed him to seeking these elusive subjects, but only that in fact it did lead him to do so. Yet his analysis would collapse only if, either by itself or in conjunction with other true propositions, it actually *entailed* the objectionable conclusion. Now 'Something is a horse' does not, either by itself or in conjunction with any true propositions I can think of, entail 'This or that logical atom (or whatever an ultimate subject of discourse may be) is a horse'. In trying to establish a connexion between the two Miss Macdonald is driven to suggest that a patently inappropriate question is natural. If someone should say 'Something is a horse', it would be inappropriate to ask 'What is the something that is a horse?' One might, it is true, have been inclined to ask this if, owing to its unfamiliar idiom, one had misunderstood what was said. But not otherwise. For no particular something ('*the* something') would have been mentioned of which one might

[1] This is a mistake. Lord Russell's analysis would be '"If x is equine then x neighs" is always true', the connective 'if . . . then . . .' being interpreted as a material implication.

ask 'Is it this or that?' or 'What is it?' Lord Russell's point is that 'Something' is not a subject in the usual sense: it does not stand for any particular object about which a statement is made. Existential prospositions are general, not singular.

Miss Macdonald's further objections appear to me to be of the same order: good reasons are presented for rejecting theories with which Lord Russell's analysis is historically associated, and which its author has at some time or other countenanced, but from which it is logically distinct. Two, to which Miss Macdonald particularly objects, are the theories that proper names and demonstrative pronouns denote logical atoms, and that classificatory nouns like 'man', 'horse', 'god' function in all respects as do descriptive adjectives like 'tall', 'piebald', and 'omnipotent'.

What 'the important differences' are 'between existence propositions and those which ascribe predicates to subjects' Miss Macdonald does not profess to know. Tentatively, she denies that existential propositions 'ascribe their defining properties' (presumably the defining properties of what they assert to exist) 'to subjects' (*loc. cit.* p. 91). This is ambiguous. It may mean that existential propositions have no definite subject, *i.e.* that they are general, which is Lord Russell's position; or it may mean that the grammatical subject of the verb 'to exist' does not function predicatively. Miss Macdonald does not explicitly recognize the former as an alternative. 'Nor does it follow,' she concludes, 'that existence propositions have no subjects. Their subjects are things which are asserted to exist' (*ibid.*, p. 91). Even for the examples which we have examined, this conclusion cannot, I believe, be maintained; applied to negative existential propositions, it is astonishing. 'Ghosts do not exist' would have ghosts for its subject, that is, it would be about ghosts. But how can it be about ghosts, if it says that there are no ghosts for it to be about? It is inconsistent to accept Kant's contention that existence is not a predicate, which (*ibid.*, p. 89) Miss Macdonald does, and to reject its consequence that existential propositions do not characterize, are not about, their apparent subjects. 'Lions exist,' as Kant showed, does not characterize, or say anything about, or ascribe any predicate to lions: it asserts that lions are a subject about which characterizing statements may be made. So 'Ghosts do not exist' does not assert that ghosts lack a certain character, but that ghosts are not a subject about which characterizing statements may be made. It is because they 'posit' a certain sort of thing as a subject, or deny that it may be one that exist-

ential propositions cannot make characterizing statements about subjects *already* posited. Whether or not Lord Russell has succeeded in his attempt at an exact and technical account of Kant's discovery, to dismiss the discovery itself is philosophically suicidal.

Mr. Martin Shearn, in his recent paper *Russell's Analysis of Existence* (ANALYSIS XI, p.124), contrives to overlook a considerable body of work accomplished by Lord Russell and philosophers who have learned from him. Lord Russell's 'definition of existence', as we have observed, asserts that what the sentence 'Lions exist' states is less misleadingly expressed by such unidiomatic sentences as 'Something is a lion', or 'At least one value of the function "x is a lion" states a truth.' Mr. Shearn has noticed that there are many expressions, among them 'is feared', 'is worshipped', 'is talked of', 'is imagined', 'is thought of', 'is dreamed of', 'is believed in', 'is prayed to' which have this curious property: it may be true to say both that Moloch is feared (worshipped, talked of, imagined, believed in, thought of, etc.) and that Moloch does not exist. Lord Russell's blindness to the significance of these expressions may be excused, because they are 'unlike the functions . . . which occur in Logic books' (*loc. cit.*, p. 127). If Moloch is feared, something is feared; it may be true both that Moloch is feared and that Moloch does not exist; 'Something is feared', therefore, does *not* mean 'Something feared exists', although it appears to follow from Lord Russell's analysis that it does.

It is simply not true that Lord Russell and 'the majority of logicians' who have 'followed (him) into error' (*loc. cit.* p. 126) have ignored expressions of the kind to which Mr. Shearn invites our attention. It would be tedious to mention well-known studies of fictional statements and imaginary objects; and superfluous to point out that 'is feared', 'is worshipped', 'is prayed to' resemble 'is believed to exist', and 'is imagined'. In an important paper, *Systematically Misleading Expressions* (published in *Proceedings of the Aristolian Society* 1931–2, and recently in *Logic and Language*) Professor Ryle discriminated ordinary predicates from what he called 'quasi-ontological' predicates.[1] Just as 'exists' is not a predicate, and its grammatical subject not a logical subject, so, Professor Ryle showed, 'is a fiction', 'is an imaginary object', 'is a mere idea', 'is an abstraction' and many others are not predicates, and their

[1] I understand that Professor Ryle now thinks it would have been better to call such expressions 'ontological quasi-predicates'. This emendation does not need to be commended, and I have adopted it in the sequel.

grammatical subjects not logical subjects (*Logic and Language*, p. 17). In Lord Russell's system, it is impossible to argue 'If Mr. Churchill is existent, then something is existent, therefore an existent exists', because 'existent' is not a genuine predicate; in the same way it is impossible to argue 'If Pegasus is a fiction, then something is a fiction, therefore a fiction exists'. The point may be made in Lord Russell's symbolism. Ontological quasi-predicates like 'is a fiction', 'is imaginary', and so on cannot be represented by the variable 'F'. The argument 'If Pegasus is a fiction, then something is a fiction' is *not* of the form 'If Fa, then (∃x) Fx'. The key to Mr. Shearn's difficulties —and to the perversity of Logic books in treating of functions like 'x is human' rather than those like 'x is imaginary'—is not the range of the variable 'x', but that of the variable 'F'.[1]

What is interesting and admirable in Mr. Shearn's selection of examples is that some of them are not unequivocally quasi-predicative. In 'Moloch was feared by the Tyrians', as used in any ordinary modern context, 'was feared' is quasi-predicative: the whole sentence states only that the Tyrians thought that a certain god existed, who was called Moloch, and who had frightening habits. In 'Dr. Arnold was feared by his pupils,' 'was feared' is genuinely predicative: it states that Dr. Arnold had a certain relational property. Each statement of the latter kind entails a statement of the former kind: if Dr. Arnold had the relational property of being feared by his pupils, then his pupils thought that a person existed who was called Dr. Arnold and who had frightening habits. But the converse does not hold. Mr. Shearn's argument (ANALYSIS XI, p. 125), '"Mumbo-Jumbo is feared" entails the truth of "Something is feared"', does not hold, if 'Something is feared' is interpreted as meaning that something has the relational property of being feared. It holds only in the sense—irrelevant to Lord Russell's analysis—that somebody believes that something frightening exists. Only if the two uses are confounded is it possible, and plausible, to argue that Lord Russell's analysis fails. Mr. Shearn has been less than just to Lord Russell (and to 'the majority of logicians'), even though he has drawn attention to puzzling examples.

A less important part of Mr. Shearn's paper is devoted to

[1] It would be unfair to Mr. Shearn not to mention that, in Professor Quine's symbolism, this is not so; and that Professor Quine shares his concern over 'x is feared'. (v. *Designation and Existence* in Feigl and Sellars' *Readings in Philosophical Analysis* pp. 46, 48). Professor Quine would attempt to escape Mr. Shearn's argument by denying that e.g. 'Mumbo-Jumbo' is a name, and asserting that it is 'syncategorematic and states nothing' (*ibid.* p. 48).

Lord Russell's restrictions on the range of the variables 'x', 'y', 'z'. To Lord Russell's proposal that only proper names should be values, Mr. Shearn objects that 'Mumbo-Jumbo', 'Mr. Pickwick' and other fictional proper names must be included, although '(Lord Russell) obviously intended to debar from being values of (the variables 'x', 'y', 'z') all things which do not exist. But this . . . he could have no right to do unless he had given a prior analysis of "to exist"' (*loc. cit.*, p. 128). It is clear from *An Introduction to Mathematical Philosophy* that Lord Russell thought that his account of names gave him that right. A name is 'a simple symbol designating an object' (p. 174). If it is allowed that there may be symbols which directly designate objects, his analysis holds. There is no need to modify his account of names by adding 'non-existent objects being excluded', for non-existent objects are not a species of object. Expressions which appear to designate such objects have no *designata* at all. They name nothing, and so are not genuine names. We do, it is true, say that the name 'Mr. Pickwick' designates a non-existent object; but, as Lord Russell has shown us, this is merely a way of speaking: it would be less misleading to say 'There is a fiction that "Mr. Pickwick" designates somebody, in fact it does not'.

It is curious, too, that Mr. Shearn, in a fairly lengthy review of possible restrictions on the range of the variables 'x', 'y', 'z', should have overlooked the proposal to allow only demonstrative pronouns used indicatively (*i.e.* with a pointing gesture or its equivalent, and not as referring to something already mentioned or described). Professor Moore, in his contribution to the Symposium *Is Existence a Predicate?* (*Proceedings of the Aristotelian Society*. Supp. Vol. XV) employed this device successfully. And, in a note to *Mysticism and Logic*, dated 1917, Lord Russell himself wrote, 'I should now exclude "I" from proper names in the strict sense, and retain only "this"' (p. 224).

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IN WHAT SENSE IS MODAL LOGIC MANY-VALUED?

By A. N. PRIOR

IT is often contended that the " many-valued " propositional calculi developed by Lukasiewicz and others may find application in the logic of modality, and conversely, that the logic of modality cannot be satisfactorily studied unless we use a many-valued calculus for its formal expression. On the face of it this is a curious contention, for being necessary, being impossible and being neither are surely not alternatives to being true and being false. For a proposition, whatever its modality, is always *also* true or false. (If it is necessary, it will be true ; if impossible, false ; and if neither, it may be true and it may be false but is still bound to be one or the other). These qualifications of propositions are simply not out of the same box. It is, nevertheless, possible to see a sense in which modal logic is many-valued ; and this sense is what I propose here to elucidate.

A convenient starting-point for this elucidation is a certain theorem propounded by the Polish logician Lesniewski, which we may, following Lukasiewicz, express by the formula $CK\phi pN p\phi q$. Here " C " means " If ", " K " means " Both ", " N " means " not " and " ϕp " represents any truth-function whatever containing " p " as an argument, *e.g.* p itself, Np, Cpq, Cpp, Kpq, Kpp, NCpq, CpNp, CKpqr, etc. ; so that the formula means, roughly, " If any truth-function ϕ holds both for a proposition p and for its negation Np, then it holds for any proposition q whatever ". This is plainly true, and depends for its truth on what we may call the two-valued character of the logic of truth-functions. For the truth-value of any truth-function depends solely on the truth-values of its arguments, and there are no other truth-values beside the pair of which one will be possessed by p and the other by Np ; so that any proposition q must either have the same truth-value as p or the same truth-value as Np, and any truth-function which is true when one of its arguments has the same truth-value as p because it *is* p, and also true when that argument has the same truth-value as Np because it *is* Np, will be true when that argument has any truth-value whatever, and so will be true when that argument is any proposition whatever. (For example, letting ϕp be p, the formula yields CKpNpq, " If both p and not p then q "). But it has been pointed out by Lukasiewicz[1] that if

[1] See I. M. Bochenski, *La Logique de Théophraste* (1947), pp. 99–100 and n. 289 ; J. Lukasiewicz, *On Variable Functors with Propositional Arguments*, Proceedings of the Royal Irish Academy, 54 A 3 (1951), §1.

we let ϕ be the modal operator " It is contingent that ", in the Aristotelian sense of " It is neither necessary nor impossible that ", we obtain quite fantastic results, and equally fantastic results are obtainable by letting ϕ be the simple " It is possible that ".[1] It can hardly be contended that if in any case it is both possible that p and possible that not p, then anything whatever is possible. And we may express this if we like by saying that the " principle of bivalence " (which we may regard as being expressed by the formula $CK\phi p\phi Np\phi q$) does not hold for modal operators. But what is the point of expressing this fact in this manner?

Let us consider the analogous case of quantifiers. If the logic of quantification had a law analogous to $CK\phi p\phi Np\phi q$ it would be $CK\Phi xfx\Phi xNfx\Phi xgx$, where " Φx " is any quantifier. But this is plainly false if we let our Φx be the existential quantifier Ex—it is no law of logic that if it is true of any property both that something has that property and something lacks it, then something has any property whatever. That is, in some similar sense to that in which modal logic is multi-valued, the logic of quantification is multi-valued too. And what that sense is in this case becomes quite clear when we consider the different notions of " equivalence ", *i.e.* " having the same value ", which are of immediate relevance in the logic of truth-functions on the one hand and the logic of quantification on the other. Two propositions are truth-functionally (" materially ") equivalent if they are either both true or both false; and if we have any proposition p and its negation Np, then any proposition whatever is " equivalent " in this sense either to the one or to the other. But in the logic of quantification we have to do with the equivalence not of propositions but of predicates, *i.e.* with " formal " equivalence in the sense of *Principia Mathematica*. Two predicates are in this sense equivalent if each applies to every subject to which the other does; and it is *not* the case that any predicate whatever must be in this sense equivalent either to a given predicate or to the contradictory predicate. That is why $CK\Phi xfx\Phi xNfx\Phi xgx$ cannot be asserted as a law. Presupposed by this different sense of " equivalence ", *i.e.* of " having the same value ", is a different sense of " value ". The " values " of a predicate are not truth and falsehood, but its applying to this and only this, or that and only that, set of subjects (" applies to no subject " being one possible " value " in this sense). There are of course not merely two but an

[1] Cf. my paper *On Propositions Neither Necessary nor Impossible* (to appear in the Journal of Symbolic Logic).

indefinite number of sets of subjects which may thus form the complete set of subjects to which a predicate applies; and it is in this sense that the logic of quantification is many-valued. It is *not* meant by this that the logic of quantification operates with other *truth*-values beside the usual two; the point is rather that the "values" which are of immediate relevance when we are employing quantifiers are not truth-values at all. The "values" of quantified predicates are in fact their extensions.

The "values" of quantified predicates are, however, related to truth-values in at least two ways. In the first place, to say that a, b and c are the subjects to which and to which alone the predicate f applies, is in effect to say that the propositions fa, fb and fc are true, while fd, etc. are false. And in the second place, to say that two predicates are "equivalent" with respect to their quantification, *i.e.* have the same extension, is to say that any proposition in which either of them is quantified in any way will have the same truth-value as that in which the other is quantified in the same way. If f and g are formally equivalent predicates, *i.e.* apply to the same set of subjects, then *E*xfx ("Something is F") will have the same truth-value as *E*xgx, *Π*xfx ("Everything is F") the same truth-value as *Π*xgx, *Π*xCfxhx ("If anything is F it is H") the same truth-value as *Π*xCgxhx, etc. (Material equivalence functions in truth-functional logic in a similar way. If p and q are materially equivalent, then Np will have the same truth-value as Nq, Cpr as Cqr, and so on).

We may now return to modal operators, and deal with them similarly. We may say that p and q are "modally equivalent" propositions, *i.e.* that they have the same "modal value", if any proposition in which either is modally qualified in any way will have the same truth-value as that in which the other is modally qualified in the same way. For example, if p and q are modally equivalent, then Mp will have the same truth-value as Mq, C'pr will have the same truth-value as C'qr (where C'pr means "P strictly implies R", *i.e.* NMNCpr), and so on. And just as predicates are formally equivalent when and only when each formally implies the other, so propositions are modally equivalent when and only when each *strictly* implies the other. It is not so easy to find a sense for "p has the modal value x" as it is to find a sense for "p has the same modal value as q"; but we might say that the modal value of a proposition is the set of possible states of affairs in which, and in which only, the proposition in question is true.[1] This gives us another relation between

[1] Alternatively, we might copy the procedure of *Principia Mathematica* in defining numbers, and say that the modal value of a proopsition is the class of propositions modally equivalent to it.

modal values and truth-values beside that involved in the definition of modal equivalence, just as there is another relation between " quantificational values " and truth-values beside that involved in the definition of quantificational or formal equivalence. And there are a multiplicity of modal values (or modal extensions, as we might also call them), just as there are of quantificational values.

It is characteristic of quantifiers that when attached to predicates (with variables inserted in the appropriate places) they form propositions. That is why they effect a correlation between quantificational values and truth-values—the truth-value of a proposition formed by quantifying a predicate depends on the quantificational value of that predicate. But there are also operators upon predicates which form other predicates, in such a way that the quantificational value of the constructed function depends on the quantificational value or values of its argument or arguments. Thus the set of subjects to which, and to which alone, the compound predicate " both f and g " applies, is determined by the sets of subjects to which, and to which alone, f applies and g applies. And we may represent this relation by something like a truth-table. Let us suppose for simplicity that there are only two objects in the universe. This will not mean that there will only be two possible " quantificational values " for predicates in this universe. For from two objects we obtain four sets of objects—the zero set, the set consisting of the first object only, the set consisting of the second object only, and the set consisting of both. The possible quantificational values will therefore be " applying to neither " (call this " 1 "), " applying to the first only " (call this " 2 "), " applying to the second only " (call this " 3 ") and " applying to both " (call this " 4 "). We may then construct the following " quantificational value-table " for " Both f and g ", the possible values of f being listed down the side and the possible values of g at the top :

	1	2	3	4
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
4	1	2	3	4

From the first horizontal row under the line, we learn that if f applies to no object (*i.e.* has the value 1), then no matter what g applies to, " Both f and g " applies to no object (*i.e.* has the

value 1). From the second horizontal row we learn that if f applies to the first object only (has the value 2), then if g applies to no object or to the second object only, " Both f and g " applies to no object, while if g applies to the first object only or to both objects, " Both f and g " applies to the first object only. And so on with the others. The corresponding table for " Either f or g " (non-exclusive) would be as follows :—

	1	2	3	4
1	1	2	3	4
2	2	2	4	4
3	3	4	3	4
4	4	4	4	4

These are just like the four-valued matrices which one encounters in such writers as Lukasiewicz, and we have here a possible interpretation for such matrices.

In the case of ordinary modal functions, although both the function and its argument or arguments are propositions, there is a similar heterogeneity to that which we have with quantifiers —it is the *truth*-value of the modal function which depends on the *modal* value or values of its argument or arguments.[1] There are also functions of which the *modal* value depends on the modal value or values of their arguments, though these are not themselves modal functions but ordinary truth-functions. Let us again simplify matters, and suppose that there are only two possible states of affairs. This gives rise to four modal values that a proposition may have—it may be true in neither state of affairs (Value 1), true in the first but not the second (Value 2), true in the second but not the first (Value 3) and true in both (Value 4). (If true in neither, it will have a modality of impossibility in this " modal universe " ; if true in both, one of necessity). We may then construct the following " modal value table " for the truth-function " Both p and q ", the possible modal values of p being listed down the side and those of q along the top :

[1] On their modal value, be it noted ; not, except in some cases and consequentially, on their modality. Thus if P and Q both have a modality of possibility, this does not suffice to determine whether or not P strictly implies Q. That depends on whether or not the possible states of affairs in which P is true are wholly included in those in which Q is true ; and this in turn depends on what *are* the possible states of affairs in which P and Q are respectively true, i.e. on their modal values. It is true that if P has a modality of impossibility, P will strictly imply Q whatever Q may be, and so whatever may be its modal value ; but this is because (*a*) all impossible propositions have the same modal value, and (*b*) that modal value consists in being true in *no* possible state of affairs, so that there is no possible state of affairs in which P and not-Q are both true.

	1	2	3	4
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
4	1	2	3	4

Here the first horizontal line tells us that if p is an impossible proposition, " Both p and q " will be impossible no matter what the modal value of q may be; the second line tells us that if p is true in the first possible state of affairs only, then " Both p and q " will be impossible if q is either impossible or true in the second possible state of affairs only, and will be true in the first possible state of affairs only if q is either necessary or true in the first possible state of affairs only; and so on with the rest. This value-table, it will be seen, has the same form as the quantificational value-table for the compound predicate " Both f and g ", and we have here and in similar cases another possible interpretation for four-valued matrices.

We may also construct value-tables for quantifications and for ordinary modal functions, but in such tables different sorts of " values " will be involved on the borders and in the body of the table. Thus if there are no objects but A and B, and " 1 " means applying to neither, " 2 " applying to A only, " 3 " applying to B only, and " 4 " applying to both, we may construct the following for *Π*xCfxgx (" If anything is F it is G "):

	1	2	3	4
1	T	T	T	T
2	F	T	F	T
3	F	F	T	T
4	F	F	F	T

Here the values indicated on the borders are quantificational values and those in the body of the table are truth-values; the first horizontal row tells us that if f applies to neither A nor B, *Π*xCfxgx is true no matter what g applies to; the second row tells us that if f applies to A only, then *Π*xCfxgx is true if g applies to A only or to both, and false otherwise; and so on. The same table, with 1, 2 3 and 4 representing modal values, would exhibit the properties of C'pq in a modal universe with only two possible states of affairs. But this is not the sort of matrix which one generally encounters in many-valued logics, and there is quite obviously no suggestion in such matrices of there being more than the ordinary two *truth*-values.

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MESSRS. BLACK AND TAYLOR ON TEMPORAL PARADOXES

By ADOLF GRÜNBAUM

1. IN recent issues of this journal (vol. XI, pp. 92–101 and vol. XII, pp. 38–44), Messrs. Black and Taylor have questioned anew the adequacy of rival solutions of Zeno's paradoxes of motion.

I wish to take issue with both writers and shall endeavour to show that neither of them has been successful in dealing with the Zenonian challenge as construed by A. N. Whitehead[1] and, more recently, by Findlay[2]: how can any motion or time-interval be meaningfully resolved into elements which form a mathematically dense set, *i.e.*, which are ordered such that no event has an immediate temporal or spatial successor? It is incumbent upon both Mr. Black and Mr. Taylor to deal with this version of the problem, since each of these authors has made claims which, as we shall see, make such an encounter logically inescapable.

Though contending that their handling of the Zeno issue is unsatisfactory, I shall *not* maintain that the argument which Whitehead regarded as the viable content of the Eleatic's paradoxes still constitutes an open problem. For in a recent publication elsewhere,[3] I have given a non-Whiteheadian refutation of this argument by providing a *semantical justification* for postulating physical motion and change to be a linear Cantorean continum (dense set) of point-events. And it is because Russell's Cantorean treatment of Zeno's puzzles[4] lacked such a semantical justification that I think it has rightly been judged inadequate, as is evidenced by the continued appearance of rebuttals to Zeno.

2. Mr. Black's particular retort takes the form of a denial of the physical relevance of the infinities encountered in the mathematical theory of motion and of an affirmation of a complete finitism for the actual process of motion. He believes that he can couple his emphasis on the finitist character of all that Achilles does physically with the acceptance of the non-finitary mathematical theory of motion by claiming that the latter is only "one way in which we find it useful to *describe* the physical reality". (p. 101) Mr. Black's paper contains no statement averring that he could furnish a satisfactory reconstruction of the mathematical theory of motion on the basis of a relational theory of space à la Leibniz in which distances are

[1] A. N. Whitehead, *Process and Reality* (New York, 1929), pp. 95, 105–107.

[2] J. N. Findlay, "Time: A Treatment of Some Puzzles," *Essays on Logic and Language*, ed. with an introd. by A. G. N. Flew (Oxford and New York, 1951), pp. 49–50.

[3] A. Grunbaum, "Relativity and the Atomicity of Becoming," *The Review of Metaphysics*, vol. IV (1950), pp. 143–186.

[4] B. Russell, *Our Knowledge of the External World* (London, 19194), chs. V–VII.

intensive magnitudes.[1] Thus, we cannot call upon him to demonstrate the success of such an undertaking. But his finitist view is untenable unless he does treat distances as intensive magnitudes, *i.e.*, as positive magnitudes which do *not* presuppose all the constituents of which they can be regarded to be the sum. For otherwise to accept the linear Cantorean continuum as a basis for describing the motion while refusing to say that the runner traverses a dense infinity of points which are both ordinally and cardinally a continum is to wish to eat one's cake and have it. The very analytic geometry which is used in the mathematical theory of motion commits Mr. Black to the ordinal isomorphism between the real numbers and extensionless *spatial* elements on the number axis (via the Archimedean and Cantor-Dedekind axioms). Yet he wishes to be held only to the *arithmetic* terminus of this isomorphism while imposing an *ad hoc* prohibition on taking the *geometric* terminus seriously as well. In the absence of a satisfactory theory of space-quantization to the contrary,[2] the geometrical existence of a continuum of points both between the end-points of each step taken by the runner and on the diameter of each atom on the latter's path can therefore not be consigned to the category " in thought only ". Moreover, " To divide a thing ' in thought ' " is not, as Mr. Black would have it, " merely to halve the *numerical* interval which we have assigned to it " (p. 101, italics supplied); it is also to realize that a *geometrical* mid-point lies half-way between the two end-points. To assign the number 1/2 to each of the halves of the egg considered by Mr. Black is not, to be sure, to suppose that we " must make some special effort to hold its halves together " (p. 101). But it is to become aware that if someone ate the egg, for instance, he would be deriving nourishment from each of its two halves just as unfancifully as he would be nourished by the whole ! Mr. Black's analysis here is vulnerable to the telling objection which W. D. Ross urged against Aristotle's doctrine of the potential rather than actual existence of points of division : " . . . It surely cannot be maintained that a moving particle actualizes a point of space by coming to rest at it. It can come to rest only at a point that is there to be rested at. And when it does not rest but moves continuously, the pre-existence of the points on its course is equally pre-supposed by its passage through them . . ."[3] Finally, since Mr. Black evidently reserves the term " thing " for the objects of positive metrical

[1] Cf. B. Russell, *The Philosophy of Leibniz* (London, 1937), pp. 112–114 and *Principles of Mathematics* (New York, 1903), ch. XXXI; pp. 180, 413.

[2] For some of the difficulties besetting theories of space-quantization, see H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton, 1949), p. 43.

[3] W. D. Ross, *Aristotle's Physics* (Oxford, 1936), pp. 74–75.

extension encountered in macroscopic experience, it is trivial for him to tell us that the spatial points (and pointevents) of the mathematical theory of motion are not spatio-temporal things. The same must be said of his assertion that " all the things he [the runner] really does are finite in number " (pp. 100–101) in virtue of the meanings which he assigns to " really " and " does " in this context. It would seem, therefore, that Mr. Black's endorsement of the mathematical theory of motion requires him to say after all that the runner does do infinitely many things. For on that theory of motion, the runner must be held to traverse a continuum of points in each of the finite number of steps he takes. The alternative is to attempt a mathematically successful supplantation of Cantor's view that " in order to achieve a more satisfactory *description of nature*, the ultimate or genuinely *simple* elements of matter must be postulated as *actually infinite in number* and . . . *spatially* they must be regarded as *entirely without extension* ".[1] Pending Mr. Black's justification of his repudiation of Cantor's view, I must conclude that (1) Mr. Black is mistaken in asserting that " We create the illusion of the infinite tasks by the kind of mathematics that we use to describe space, time and motion." (p. 101), (2) His finitist solution of Zeno's problem is unsuccessful and he must face that problem in the Whiteheadian form given above. Consequently, I can give assent to Mr. Taylor's strictures (pp. 41–42) on Mr. Black in so far as these are directed against Mr. Black's argument from finitism.

3. Mr. Taylor tells us that Mr. Black's polemic against the concept of an infinite number of distinct acts is vitiated by the latter's alleged tacit demand that a discrete denumerable infinity of terms should have a last term, a demand whose realization is, of course, logically impossible.[2] Instead of laying down this self-contradictory requirement, however, Mr. Black actually seems to be making the following *synthetic* assertion: If a physical process does consist of a series of distinct consecutively-ordered operations and if this process is to be completed in a finite time, then such " completion " can be achieved only by the existence and performance of a *last* operation in the series. Thus, if I understand him correctly, Mr. Black regards the impossibility of completing such a process in a finite time as fundamentally *ordinal* in character and only derivatively metrical. It is this assumption on the part of Mr. Black and other writers which generates the question of *how* the process can be completed if indeed there is no last operation in the series. Mr. Taylor's charge of tacit self-contradiction has therefore missed its mark

[1] Georg Cantor, *Gesammelte Abhandlungen*, ed. E. Zermelo, (Berlin, 1932), p. 275. Italics are Cantor's.

[2] Cf. Leibniz' argument against John Bernoulli quoted in H. Weyl, *op. cit.*, p. 44.

and he must rest his case entirely on the claim that Mr. Black's assumption is either *physically* false or gratuitous. And Mr. Taylor advances this claim by maintaining that there can well be an actual infinity of distinct, consecutively-ordered operations, provided only that they satisfy the *metrical* condition of being of appropriately decreasing size. Thus he denies Mr. Black's contention concerning the indispensability of a last operation. To substantiate this rival *synthetic* proposition, Mr. Taylor unfortunately relies entirely on arguments drawn from the domain of the *definitions* of abstract arithmetical theory and thus argues from definitions to conclusions about matters of fact, as I shall now show.

The extension of the concept of " sum " from the case of a finite number of finite cardinals to that of a discrete denumerable sequence of such cardinals is a matter of *definition*.[1] The familiar arithmetic theory of infinite series employs a consistent generalization of the concept of finitary sum in which the arithmetic " sum " of a denumerated discrete series of finite cardinals is *defined* as the arithmetic limit of a sequence of partial arithmetic sums of terms of the series. But an alternate arithmetic theory could quite consistently define the " sum " of an infinite series of finite cardinals as *infinite* even in the case of a converging series. It is now clear that the issue between Mr. Taylor and Mr. Black centers on the following *empirical* question: Which of these two arithmetic theories is physically relevant when given a semantical interpretation in terms of the intervals of the race ? Mr. Russell[2] and Mr. Taylor are accustomed to the limit definition of the sum of an infinite series of finite cardinals and are thus led into taking it for granted that the particular arithmetical theory employing *this* definition is the physically relevant one. Thus Mr. Taylor tells us that if there are no physical limitations upon the rapidity of our thought and speech and if we then ask how long it will take to enunciate the infinite series 1, 1/2, 1/4, . . . ,—the first act requiring one minute and each successive act taking half the time of its predecessor—the answer " would *quite obviously* be, not ' Forever ! ', but ' Two minutes ' " (p. 43 ; italics supplied). Mr. Taylor's belief that the answer to the physical question he raises follows " quite obviously " from arithmetic theory is surprising in the face of his own correct statement that " Through the summation of a geometrical progression one can specify at what point a process of the sort described by Zeno will terminate, but the question that remains, and the only one Zeno asked, was how such a *process* can be

[1] Cf. the discussion of Hankel's principle in F. Waismann, *Introduction to Mathematical Thinking*, tr. by T. J. Benac (New York, 1951), ch. 4.

[2] B. Russell, " The Limits of Empiricism," *Proc. Arist. Soc.*, n.s. 36, 144 (1935–36).

consistently conceived in the first place" (p. 38) More importantly, such authorities as H. Weyl and O. Holder seem to give support to Mr. Black on the physical issue here in question.[1] It is true, of course, that the arithmetic sum of the lengths of any *finite* number of temporal intervals 1, 1/2, 1/4, . . . , $1/2^{n-1}$ in a physical process is always less than 2. But we see from the foregoing considerations that this arithmetic fact alone does *not* entail that the arithmetic "sum" of the entire infinite series *must be* 2. It entails even less that "*no* process conceived as divisible into intervals of the series 1 minute, 1/2 minute, 1/4 minute . . . etc., *could* last forever" as Mr. Taylor would have it (p. 44).[2]

Let us grant, however, that Mr. Taylor's physical assumption is the correct one. Does this assumption ensure the removal of the difficulty raised by Zeno, James and Whitehead? It seems quite clear to me that it does not. For Mr. Taylor himself maintains or, at least, suggests quite rightly (p. 42) that a stone or the path of the runner does actually "consist of infinitely many parts". To be more precise, we should say that the path consists of a dense infinitude of points and that the process of traversing that path consists of a like infinitude of point-events. Accordingly, a *necessary condition* for the runner's succeeding in traversing the denumerable set of intervals 1, 1/2, 1/4, . . . is that he accomplish the task of traversing the *dense* set (continuum) of points constituting *each* of the intervals in this or any other series of intervals into which the path may be divided. It was precisely the possibility of satisfying this necessary condition that was called into question by James and Whitehead in response to the stimulus of Zeno. Now Mr. Taylor has claimed that the runner can accomplish the task of traversing the set of intervals 1, 1/2, 1/4, . . . On the other hand, James and Whitehead contend that the very condition which, on Mr. Taylor's own showing, is *necessary* for the realization of that task, cannot be fulfilled. I therefore conclude that if Mr. Taylor's claim is to be regarded as justified, he must show where James and Whitehead were mistaken. And I refer the reader to my earlier paper (cf. footnote 3 above) for the required criticism of the arguments of these two philosophers.

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[1] Cf. H. Weyl, *Das Kontinuum* (Berlin, 1918), p. 15, *Die Stufen des Unendlichen* (Jena, 1931), pp. 2–3, *Philosophy of Mathematics and Natural Science*, op. cit., p. 42, and O. Holder, *Die Mathematische Methode* (Berlin, 1924), pp. 98 and 539. See also A. Grünbaum, *op. cit.*, pp. 178–182.

[2] For an espousal of views supporting Mr. Taylor on this point,, see R. M. Blake "The Paradox of Temporal Process," *Journal of Philosophy*. 23 (1926), p. 652.